Name: _____

Math 10560, Final Exam:
May 5, 2008

Instructor: _____

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
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PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)	15.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)	16.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)	17.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	18.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	19.	(a)	(b)	(c)	(d)	(e)
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11.	(a)	(b)	(c)	(d)	(e)	25.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)						
13.	(a)	(b)	(c)	(d)	(e)						
14.	(a)	(b)	(c)	(d)	(e)						

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Multiple Choice

1.(6 pts.) The function $f(x) = x + e^x$ is one-to-one. Find $(f^{-1})'(1)$.

(a) $\dfrac{1}{1+e}$ (b) 1 (c) 0 (d) e^{-1} (e) $\dfrac{1}{2}$

2.(6 pts.) Solve the following equation for x:

$$\ln(x+4) - \ln x = 1 \ .$$

(a) $x = \dfrac{4}{1-e}$

(b) $x = \dfrac{4}{e-1}$ and $x = \dfrac{4}{e+1}$

(c) There is no solution.

(d) $x = e + 2$ and $x = e - 2$

(e) $x = \dfrac{4}{e-1}$

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3.(6 pts.) Find the derivative of $(x^2+1)^{x^2+1}$.

(a) $(x^2+1)^{x^2+1}\big(2x\ln(x^2+1)\big)$

(b) $(x^2+1)^{x^2+1}\ 2x\big(\ln(x^2+1)+1\big)$

(c) $2x(x^2+1)^{x^2}$

(d) $(x^2+1)^{x^2+1}$

(e) This function is not defined and hence has no derivative.

4.(6 pts.) Compute the integral $\displaystyle\int_0^1 \frac{x^2dx}{x^3+1}$.

(a) $\dfrac{\ln 2}{3}$ (b) $\ln 2$ (c) 0 (d) $\dfrac{1}{2}$ (e) ∞

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5.(6 pts.) Find $\lim_{x\to+\infty} \dfrac{3^x}{3x^3+8x^2-117x+10}$.

(a) Does not exist but is not $+\infty$ or $-\infty$.

(b) $\dfrac{1}{3}$

(c) $+\infty$

(d) 1

(e) $-\infty$

6.(6 pts.) Simplify $\tan^{-1}(\tan\frac{7\pi}{8})$.

(a) 0

(b) not enough information to tell.

(c) $\dfrac{7\pi}{8}$

(d) $-\dfrac{\pi}{8}$

(e) $\dfrac{\pi}{8}$

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7.(6 pts.) Calculate the integral $\int_0^1 xe^{-x}dx$.

(a) $1-\frac{2}{e}$ (b) 1 (c) $-1-\frac{1}{e}$

(d) $1-\frac{1}{e}$ (e) $\frac{1}{e}$

8.(6 pts.) $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^2 x \; dx =$

(a) $\frac{5}{12}$ (b) $\frac{4}{21}$ (c) $\frac{2}{15}$

(d) $\frac{2}{45}$ (e) $-\frac{2}{15}$

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9.(6 pts.) Evaluate the integral

$$\int \frac{x^2+2x-1}{x^3-x}\,dx.$$

(a) $\ln\left|x(x+1)^2\right|+C$

(b) $\ln\left|\dfrac{x^2-x}{x+1}\right|+C$

(c) $\ln\left|\dfrac{x^2+x}{x-1}\right|+C$

(d) $\ln\left|x^3-x\right|+C$

(e) $-\dfrac{1}{x^2}+\ln\left|\dfrac{x+1}{x-1}\right|+C$

10.(6 pts.) Estimate

$$\int_0^1 \cos(x^2)\,dx$$

using the Midpoint Rule with $n=4$.

(a) $\frac{1}{4}(\cos(\frac{1}{16})+2\cos(\frac{1}{4})+2\cos(\frac{9}{16})+\cos(1))$

(b) $\frac{1}{8}(\cos(\frac{1}{16})+\cos(\frac{1}{4})+\cos(\frac{9}{16})+\cos(1))$

(c) $\frac{1}{4}(\cos(\frac{1}{64})+2\cos(\frac{9}{64})+2\cos(\frac{25}{64})+\cos(\frac{49}{64}))$

(d) $\frac{1}{8}(1+\cos(\frac{1}{16})+\cos(\frac{1}{4})+\cos(\frac{9}{16}))$

(e) $\frac{1}{4}(\cos(\frac{1}{64})+\cos(\frac{9}{64})+\cos(\frac{25}{64})+\cos(\frac{49}{64}))$

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11.(6 pts.) Evaluate

$$\int_0^\infty \frac{e^x}{e^{2x}+1}\,dx.$$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{1}{2}$ (d) 1 (e) diverges

12.(6 pts.) The curve $y = 1 - x^2$ with $0 \le x \le 1$ is revolved about the **y-axis**. Find the surface area of the resulting surface.

(a) $\frac{2\pi}{15}\left(6\sqrt{3}+1\right)$

(b) $\pi\left(\sqrt{5}+\frac{1}{2}\ln|\sqrt{5}+2|-2\right)$

(c) $\frac{\pi}{6}\left(5\sqrt{5}-1\right)$

(d) $\frac{\pi}{6}\sqrt{5}$

(e) $\frac{2\pi}{35}\left(2\sqrt{3}-22\right)$

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13.(6 pts.) Find the centroid of the region bounded by

$$y = \frac{1}{x^2} \qquad y = 0, \qquad x = 1, \qquad x = 2.$$

(a) $(\bar{x}, \bar{y}) = \left(2\ln 2,\ \frac{7}{24}\right)$

(b) $(\bar{x}, \bar{y}) = \left(\frac{8\ln 2}{7},\ \frac{1}{6}\right)$

(c) $(\bar{x}, \bar{y}) = \left(\frac{3}{2},\ \frac{7}{24}\right)$

(d) $(\bar{x}, \bar{y}) = \left(\frac{3}{2},\ \frac{1}{160}\right)$

(e) $(\bar{x}, \bar{y}) = \left(\frac{1}{2\ln 2},\ \frac{7}{4\ln 2}\right)$

14.(6 pts.) Use Euler's method with step size 0.2 to estimate $y(0.4)$ where $y(x)$ is the solution of the initial-value problem

$$y' = 1 - 5xy, \qquad y(0) = 0.$$

(a) $y(0.4) \approx 0.8$

(b) $y(0.4) \approx 0.2$

(c) $y(0.4) \approx 0.36$

(d) $y(0.4) \approx 1$

(e) $y(0.4) \approx 1.5$

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15.(6 pts.) Solve the differential equation

$$y' = \sqrt{x}e^{-y}.$$

(a) $y = -\ln\left(\frac{1}{2}x^{-1/2} + C\right)$

(b) $y = \ln\left|\frac{2}{3}x^{3/2}\right| + C$

(c) $y = -\ln\left(\frac{2}{3}x^{3/2} + C\right)$

(d) $y = \ln\left(\frac{2}{3}x^{3/2} + C\right)$

(e) $y = \ln\left|\frac{1}{2}x^{1/2}\right| + C$

16.(6 pts.) Solve the differential equation

$$y' + 2y - 2e^x = 0.$$

(a) $y = \frac{2}{3}e^x + Ce^{-2x}$

(b) $y = \frac{2}{3}e^x + C$

(c) $y = 2e^{-x} + Ce^{-2x}$

(d) $y = \dfrac{1}{2e^x + C}$

(e) $y = e^x - \dfrac{e^x}{x} + \dfrac{C}{x}$

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17.(6 pts.) Find the limit of the sequence $\left\{\frac{1}{2}\ln(n^2+1) - \ln(2n+1)\right\}_{n=1}^{\infty}$.

(a) Sequence diverges (b) $-\ln 2$ (c) 0

(d) $\ln 2$ (e) -2

18.(6 pts.) Determine whether the follow sequence is convergent or not. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{3^n + 4^n}{7^n}$$

(a) Series diverges (b) Series converges to $\frac{12}{7}$

(c) Series converges to $\frac{25}{12}$ (d) Series converges to $\frac{5}{12}$

(e) Series converges to $\frac{1}{7}$

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19.(6 pts.) Which is the only statement that is true about the four series

$$\text{(I)} \sum_{n=1}^{\infty} \frac{1}{n^{5/3}} \qquad \text{(II)} \sum_{n=1}^{\infty} 5^{n/3} \qquad \text{(III)} \sum_{n=1}^{\infty} \frac{1}{n^{3/5}} \qquad \text{(IV)} \sum_{n=1}^{\infty} \frac{1}{5^{n/3}} \ ?$$

(a) All four series converge

(b) (I) converges, (II), (III) and (IV) diverge

(c) (I) and (II) converge, (III) and (IV) diverge

(d) (I) and (IV) converge, (II) and (III) diverge

(e) (I) and (III) converge, (II) and (IV) diverge

20.(6 pts.) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-5)^n}{n}.$$

(a) $(4, 6)$ (b) $[4, 6)$ (c) $[-1, 1)$

(d) $(4, 6]$ (e) $(-1, 1)$

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21.(6 pts.) What is the coefficient of $(x-1)^2$ in the Taylor series of $f(x) = x^{1/3}$ centered at $a = 1$?

(a) $-\frac{2}{9}$ (b) $\frac{1}{9}$ (c) $\frac{2}{9}$ (d) $-\frac{1}{3}$ (e) $-\frac{1}{9}$

22.(6 pts.) Which of the following is the first few terms of the Maclaurin series for

$$\int \frac{\sin x}{x}\, dx \ ?$$

(a) $C - \frac{x^2}{3(3!)} + \frac{x^4}{5(5!)} - \frac{x^6}{7(7!)} + \cdots$

(b) $C + x - \frac{x^3}{3(3!)} + \frac{x^5}{5(5!)} - \frac{x^7}{7(7!)} + \cdots$

(c) $C - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots$

(d) $C + x - \frac{x^2}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

(e) $C + x + \frac{x^3}{3} - \frac{x^5}{5} + \cdots$

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23.(6 pts.) The arc length of the parameterized curve $x = \cos^2 t$, $y = \sin t$, for $0 \leq t \leq \frac{\pi}{3}$, is given by which one of the following integrals?

(a) $\displaystyle\int_0^{\frac{\pi}{3}} \cos t\sqrt{4\sin^2 t + 1}\; dt$

(b) $\displaystyle\int_0^{\frac{\pi}{3}} \sin t\sqrt{4\sin^2 t + 1}\; dt$

(c) $\displaystyle\int_0^{\frac{\pi}{3}} \sqrt{\cos^4 t + \sin^2 t}\; dt$

(d) $\displaystyle\int_0^{\frac{\pi}{3}} \sqrt{1 + \cos^2 t}\; dt$

(e) $\displaystyle\int_0^{\frac{\pi}{3}} \cos^2 t\sqrt{\cos^4 t + \sin^2 t}\; dt$

24.(6 pts.) Find the slope of the tangent line to the curve $r = 3\sin\theta$ at $\theta = 0$.
Hint: A polar curve is also a parametrized curve.

(a) -1 (b) 1 (c) 0 (d) 2 (e) π

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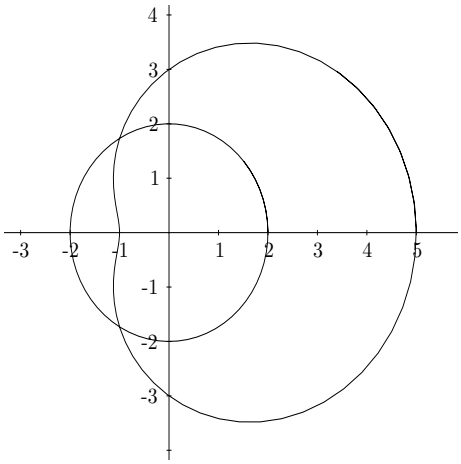
25.(6 pts.) The area inside the curve $r = 3 + 2\cos\theta$ and outside the circle $r = 2$ is given by which integral below?



Hint: The region is symmetric with respect to the x-axis.

(a) $\displaystyle\int_0^{\frac{2\pi}{3}} \left(12 + 5\cos\theta + 4\cos^2\theta\right) d\theta$

(b) $\displaystyle\int_0^{\frac{\pi}{3}} \left(5 + 12\cos\theta + 4\cos^2\theta\right) d\theta$

(c) $\displaystyle\int_0^{\frac{\pi}{3}} \left(12 + 5\cos\theta + 4\cos^2\theta\right) d\theta$

(d) $\displaystyle\frac{1}{2}\int_0^{\frac{2\pi}{3}} \left(5 + 12\cos\theta + 4\cos^2\theta\right) d\theta$

(e) $\displaystyle\int_0^{\frac{2\pi}{3}} \left(5 + 12\cos\theta + 4\cos^2\theta\right) d\theta$

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