A puzzler about sets

There are two kinds of adjectives:

- **autological**: autological adjectives describe
  themselves. Examples: "pentasyllabic" is autological,
  because pentasyllabic is pentasyllabic — it has five
  syllables. Also, "unhyphenated" is autological, and so
  is "pronounceable"

- **heterological**: heterological adjectives *don't* describe
  themselves. Examples: "monosyllabic" is heterological,
  because monosyllabic is monosyllabic — it has more
  than one syllable. Also, "hyphenated" is heterological,
  and so is "orange" (and most other adjectives).

Let S be the set of all heterological adjectives, so "orange"
$\in S$ but "pronouncable" $\notin S$.

Question: Is "heterological" $\in S$?

Answer

What if "heterological" $\in S$? Then, by the definition of S, "heterological" is an heterological adjective, so it describes itself, so by definition of "autological" it is autological, so it is *not* in S, a contradiction.

What if "heterological" $\notin S$? Then, by the definition of S, "heterological" is an autological adjective, so by definition of "autological" it describes itself, so "heterological" is heterological, so it *is* in S, also a contradiction.

$$\text{"heterological"} \in S \implies \text{"heterological"} \notin S$$
$$\text{"heterological"} \notin S \implies \text{"heterological"} \in S$$

This is the *Grelling-Nelson paradox* (1908)