Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2015.

Name:________________________________

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Practice Final Exam A

May 7, 2015

This exam contains 30 problems worth a total of 150 points. You have 2 hours to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

Some useful information can be found on pages 18 and 19.

You must record on the following page your answers to the multiple choice problems. Place an $\times$ through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)
13.	(a)	(b)	(c)	(d)	(e)
14.	(a)	(b)	(c)	(d)	(e)
15.	(a)	(b)	(c)	(d)	(e)
16.	(a)	(b)	(c)	(d)	(e)
17.	(a)	(b)	(c)	(d)	(e)
18.	(a)	(b)	(c)	(d)	(e)
19.	(a)	(b)	(c)	(d)	(e)
20.	(a)	(b)	(c)	(d)	(e)
21.	(a)	(b)	(c)	(d)	(e)
22.	(a)	(b)	(c)	(d)	(e)
23.	(a)	(b)	(c)	(d)	(e)
24.	(a)	(b)	(c)	(d)	(e)
25.	(a)	(b)	(c)	(d)	(e)
26.	(a)	(b)	(c)	(d)	(e)
27.	(a)	(b)	(c)	(d)	(e)
28.	(a)	(b)	(c)	(d)	(e)
29.	(a)	(b)	(c)	(d)	(e)
30.	(a)	(b)	(c)	(d)	(e)

Multiple Choice

1. (5 pts.) A vegetarian deli offers 5 different types of bread, 8 types of vegetables and 3 cheeses. A sandwich must have one bread and at least one cheese or one vegetable. It can have up to all 3 cheeses and all 7 vegetables. How many sandwich options does this deli offer?

(a) $5,115$ (b) $1,560$ (c) $1,275$

(d) $1,530$ (e) $32,736$

Solution:

I have 5 choices for the bread. For my cheese choices I need some subset of all of them so there are $2^3 = 8$ choices of cheese (including the empty set which means no cheese). Likewise there are $2^7 = 128$ choices of vegetables. Hence I have $8 \cdot 128 = 1,024$ choices of cheese and vegetables. *However* I cannot have no cheese and no vegetables so I only have $1,023$ choices of filling. Hence the total number of choices is $5 \cdot 1,023 = 5,115$.

2. (5 pts.) When ordering a burger at Netty's Famous Burger's in Sydney, you must first choose one type of meat from pork or beef. You then choose a subset of the seven optional fillings, tomato, lettuce, egg, bacon, cheese, pineapple and cooked onions for your burger. After you have chosen your preferred subset of fillings, you choose one sauce from the five available sauces, BBQ, Sweet Chili, Hot Chili, Mustard and Netty's special sauce. If you wish to order a burger with at least one and at most two fillings, how many different burgers are possible?

(a) 370 (b) $2,560$ (c) $2,470$

(d) 560 (e) 280

Solution:

We apply a mixture of counting techniques here. First we break the task of ordering the burger into steps and use the multiplication principle to count the number of ways of completing the task.

- Step 1: choose a type of meat $\to$ 2 ways.
- Step 2: choose a subset of at least one and at most two (either 1 or 2) fillings from the seven available $\to C(7,1) + C(7,2)$ ways.
- Step 3: choose a sauce $\to$ 5 ways.

(In step 2, we count the number of subsets of a set of size 7, with either 1 element or 2 elements. Since the set of subsets with k elements is disjoint from the set of subsets with r elements if $k \neq r$, we can use the addition principle.)

Using the multiplication principle, we get that the number of different burgers we can order with at most two fillings is

$$2 \cdot \big(C(7,1) + C(7,2)\big) \cdot 5 = 2 \cdot (7 + 21) \cdot 5 = 10 \cdot 28 = 280$$

3. (5 pts.) Brigid has 15 books and is allowed to bring at most two on vacation. How many subsets of Brigid's fifteen books have at most three elements?
Note: no books is an option.

(a) 211	(b) 1,140	(c) 1,048,576
(d) 46	(e) 121	

Solution:
We want to count the number of subsets with either 0 elements, 1 element or 2 elements. Since the set of subsets with k elements is disjoint from the set of subsets with r elements if $k \neq r$, we can use the addition principle.

- The number of subsets with 0 elements of a set of 15 elements is $C(15,0) = 1$.
- The number of subsets with 1 element of a set of 15 elements is $C(15,1) = 15$.
- The number of subsets with 2 elements of a set of 15 elements is $C(15,2) = \frac{15\cdot 14}{2\cdot 1} = 105$.

Therefore the number of subsets of a set with 15 elements which have at most three elements is $C(15,0) + C(15,1) + C(15,2) = 1 + 15 + 105 = 121$.

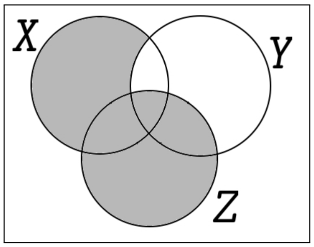
4. (5 pts.) A group of 11 alumni is visiting the Notre Dame campus and they want to have a photograph taken of them lined up in front of the Grotto. How many such photographs are possible? (A bystander will take the picture so all 11 get to be in it.)

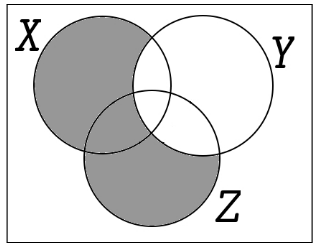
(a) 2^{11}	(b) 11^{11}	(c) $P(11,11)$
(d) $C(11,11)$	(e) 11^2	

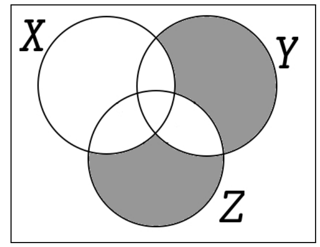
Solution:
The number of such photographs is

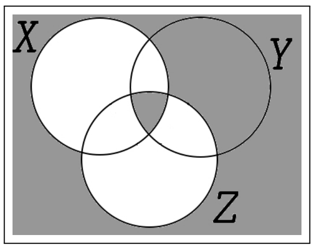
$$P(11,11) = 11! = 39,916,800$$

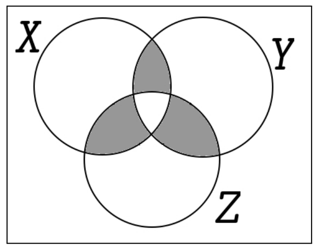
5. (5 pts.) Which Venn diagram below has $(X \cup Z) \cap Y^c$ shaded?

(a) 

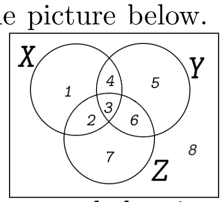
(b) 

(c) 

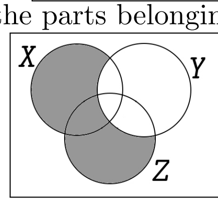
(d) 

(e) 

Solution:

You need to identify $X \cup Z$ which is the set of things in X or in Z. These are the regions numbered 1, 2, 3, 4, 6 and 7 in the picture below. Then you need to identify Y^c which consists of the regions numbered 1, 2, 7 and 8. 

You need to shade the parts belonging to both lists, namely regions 1, 2 and 7.

Hence the answer is 

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6. (5 pts.) A poker hand consists of a selection of 5 cards from a standard deck of 52 cards. There are 13 denominations, aces, kings, queens, . . . , twos, and four suits, hearts, diamonds, spades and clubs in a standard deck. On an earlier exam we saw that there are 288 poker hands which have three aces and two cards which are not aces but which are of the same denomination. How many poker hands are there with 3 cards of one denomination and 2 of another?

(a) 288 (b) 286 (c) 48

(d) $3,744$ (e) 156

Solution:

The solution for the aces problem also works if we want 3 kings, or 3 queens or 3 4's, etc. Hence there are $13 \cdot 288 = 3744$.

7. (5 pts.) The data given in the following stem and leaf plot shows the ages of all teachers at Statsville High School.

2	2	5	5	9	9	9	9		
3	0	0	5	5	7	7	9		
4	0	0	5	5	5	9	9		
5	0	0	1	4	6	7	8	8	9
6	0	1	2	3	4				

The mean age of the teachers at Statsville High is 44.2 years. What is the median age of the teachers at Statsville High?

(a) 49 (b) 45 (c) 51.5

(d) Also 44.2 (e) None of the above

Solution:

There are $7+7+7+9+5 = 35$ teachers so the median is $\dfrac{35+1}{2} = 18$ from either end. If you count carefully you get the middle 5 in the row labelled 4 so the answer is 45.

8. (5 pts.) A new disk array has six independent drives. Each disk holds a copy of the data on the other disks so all of the data can be recovered as long as one drive is still working. The array is to accompany an experiment where it will be unavailable for a year. The probability of failure within a year is 0.1 for each drive. Assuming that the failure of the various drives are independent of each other, what is the probability that at least one drive will still be working after one year?

(a) 0.999999 (b) 0.99 (c) 0.0001

(d) 0.6 (e) 0.4

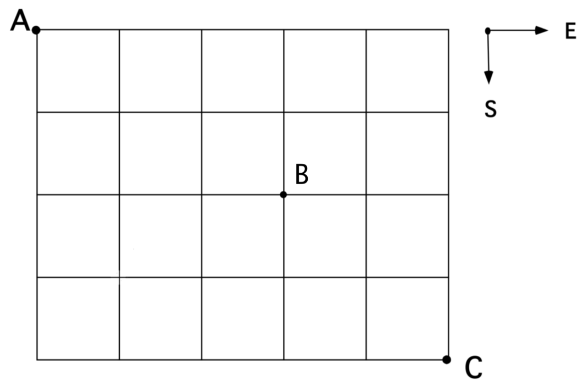
Solution:

We call the drives Drive 1, Drive 2, Drive 3, Drive 4, Drive 5 and Drive 6. We let $W_i = 0.1$ denote the probability that Drive i has failed by the end of the year. The probability that all drives have failed by the end of the first year is $\Pr(W_1 \cap W_2 \cap W_3 \cap W_4 \cap W_5 \cap W_6)$ Since the failures are independent $\Pr(W_1 \cap W_2 \cap W_3 \cap W_4 \cap W_5 \cap W_6) = \Pr(W_1)\Pr(W_2)\Pr(W_3)\Pr(W_4)\Pr(W_5)\Pr(W_6)$ and

$$\Pr(W_1)\Pr(W_2)\Pr(W_3)\Pr(W_4)\Pr(W_5)\Pr(W_6) = (0.1)^6 = 0.000001$$

. Hence the probability that at least one drive is still working is $1 - 0.000001 = 0.999999$.

9. (5 pts.) A street map of Mathland is shown below. If an Uber driver chooses a random route from A to C traveling south and east only, what is the probability that she will **not pass through the intersection at B**? (Rounded to 4 decimal places.)



(a) 0.3969 (b) 0.1746 (c) 0.5238

(d) 0.6349 (e) 0.3492

Solution:

The probability she does not pass through B, $\Pr(B^c)$, is 1 minus the probability she will pass through B, $\Pr(B)$.

$$\Pr(B) = \frac{\text{\# admissible routes through B}}{\text{Total \# of admissible routes}}$$

$$= \frac{(\text{\# admissible routes from A to B})(\text{\# admissible routes from B to C})}{\text{Total \# of admissible routes}}$$

$$= \frac{C(3+2,2)\cdot C(2+2,2)}{C(5+4,4)} = \frac{10\cdot 6}{126} = \frac{60}{126}.$$

Thus $\Pr(B^c) = 1 - \frac{60}{126} = \frac{66}{126} = 0.5238095238.$

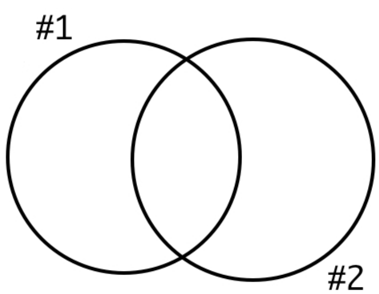
10. (5 pts.) In a Math 10120 class thirty students took Quiz 6. Twenty six of them answered the first question correctly and twenty four of them answered the second question correctly. Three students got a score of zero. How many students scored ten?

Hint: Use a Venn diagram.

(a) 24 (b) 20 (c) 22

(d) 23 (e) 18

Solution:



Let #1 denote the set of students who answered question 1 correctly.
Let #2 denote the set of students who answered question 2 correctly.

Since there are 30 students and 3 answered neither question correctly, $\#1 \cup \#2$ contains 27 students. By the inclusion-exclusion principle

$$|\#1 \cup \#2| = |\#1| + |\#2| - |\#1 \cap \#2|$$
$$27 = 26 + 24 - |\#1 \cap \#2|$$
$$|\#1 \cap \#2| = 26 + 24 - 27 = 23$$

11. (5 pts.) A sample space consists of 9 simple outcomes $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$. The probabilities are

$\Pr(x_1)$	$\Pr(x_2)$	$\Pr(x_3)$	$\Pr(x_4)$	$\Pr(x_5)$	$\Pr(x_6)$	$\Pr(x_7)$	$\Pr(x_8)$	$\Pr(x_9)$
0.04	0.06	0.08	0.13	0.13	0.15	0.17	0.22	0.02

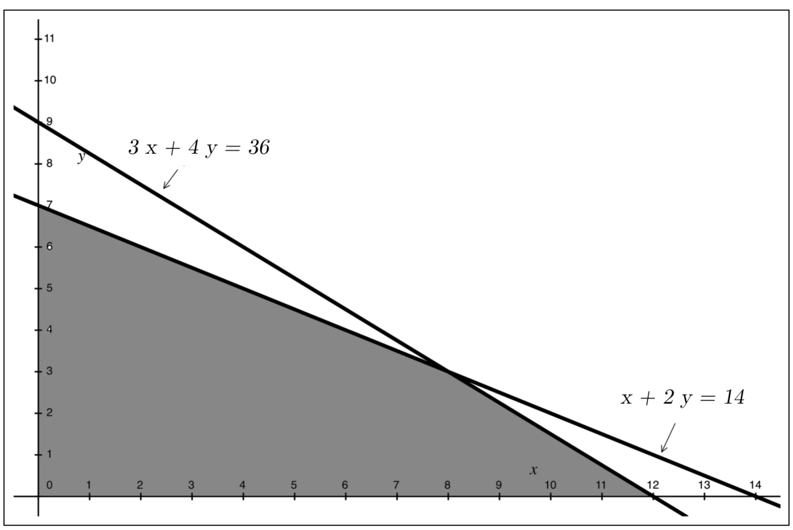
What is $\Pr(\{x_1, x_9, x_7\})$?

(a) 0.23 (b) 0.000136 (c) 0.67

(d) 0.34 (e) 0.57

Solution:
$\Pr(x_1) + \Pr(x_9) + \Pr(x_7) = 0.04 + 0.02 + 0.17 = 0.23$

12. (5 pts.) What is the maximum of the objective function $2x + 3y$ on the feasible set shown as the shaded region in the diagram below?



(a) 14 (b) 28

(c) 20 (d) No maximum of the objective function.

(e) 27

Solution:

The vertices are $(0,0)$, $(0,7)$, $(12,0)$ and the intersection of the two slant lines: $\begin{array}{rcrcr} 3x & + & 4y & = & 36 \\ x & + & 2y & = & 14 \end{array}$

$\begin{array}{rcrcr} 3x & + & 4y & = & 36 \\ 2x & + & 4y & = & 28 \end{array}$ $\quad \begin{array}{rcrcr} 3x & + & 4y & = & 36 \\ x & & & = & 8 \end{array}$ $\quad \begin{array}{rcrcr} 24 & + & 4y & = & 36 \\ x & & & = & 8 \end{array}$ so $(8,3)$.

Check values:

x	y	$2x+3y$
0	0	0
0	7	21
12	0	24
8	3	27

13. (5 pts.) A farmer has 20 acres of fields he can plant with either soybeans or corn. Each acre of corn takes 160lbs of fertilizer and 60 lbs of pesticide. Each acre of soybeans takes 80lbs of fertilizer and 120 lbs of pesticide. The farmer can get a contract to sell an acre of corn for \$2,000 per acre and an acre of soybeans for \$3,000 per acre. He has 1,600lbs of fertilizer and 1,800lbs of pesticide. The farmer wishes to maximize his net income. Suppose that C stands for acres of corn to plant and that S stands for acres of soybeans to plant. Which collection of constraints and objective functions below models this situation?

(a)
$$\boxed{\begin{array}{rcrcr} C > 0 & & S > 0 & & \\ C & + & S & \leqslant & 20 \\ 60C & + & 120S & \leqslant & 1800 \\ 160C & + & 80S & \leqslant & 1600 \\ 2000C & + & 3000S & & \end{array}}$$

(b)
$$\boxed{\begin{array}{rcrcr} C \leqslant 0 & & S \leqslant 0 & & \\ C & + & S & \leqslant & 20 \\ 160C & + & 80S & \leqslant & 1800 \\ 60C & + & 120S & \leqslant & 1600 \\ 2000C & + & 3000S & & \end{array}}$$

(c)
$$\boxed{\begin{array}{rcrcr} C \leqslant 0 & & S \leqslant 0 & & \\ C & + & S & \leqslant & 20 \\ 2000C & + & 3000S & \leqslant & 1800 \\ 160C & + & 80S & \leqslant & 1600 \\ 60C & + & 120S & & \end{array}}$$

(d)
$$\boxed{\begin{array}{rcrcr} C > 0 & & S > 0 & & \\ C & + & S & \leqslant & 20 \\ 2000C & + & 3000S & \leqslant & 1800 \\ 160C & + & 80S & \leqslant & 1600 \\ 60C & + & 120S & & \end{array}}$$

(e)
$$\boxed{\begin{array}{rcrcr} C \leqslant 0 & & S \leqslant 0 & & \\ C & + & S & \leqslant & 20 \\ 60C & + & 120S & \leqslant & 1800 \\ 160C & + & 80S & \leqslant & 1600 \\ 2000C & + & 3000S & & \end{array}}$$

Solution:

$C \geqslant 0$, $S \geqslant 0$ (you can't plant a negative amount but you want equality because planting just one crop is certainly an option).

Acreage is constrained by $C + S \leqslant 20$.

Pesticide is constrained by $60C + 120S \leqslant 1800$.

Fertilizer is constrained by $160C + 80S \leqslant 1600$.

The objective function is $2000C + 3000S$.

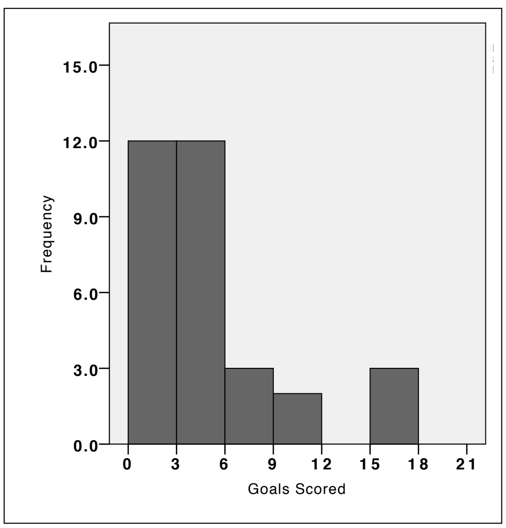
If you care, the maximum occurs at the vertex where $60C + 120S = 1800$ and $160C + 80S = 1600$ intersect which is $\left(\frac{10}{3}, \frac{40}{3}\right)$ and the farmer gets approximately \$46,666. Notice he did not plant all 20 acres.

14. (5 pts.) The number of goals scored by the 32 teams in the 2014 world cup are shown below:

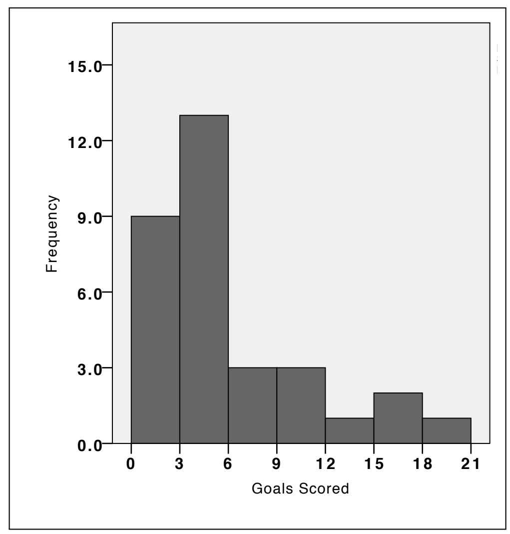
18, 15, 12, 11, 10, 8, 7, 7, 6, 6, 6, 5, 5, 5,
4, 4, 4, 4, 4, 4, 3, 3, 3, 3, 3, 2, 2, 2, 2, 1, 1, 1.

Which of the following is a histogram for the data?

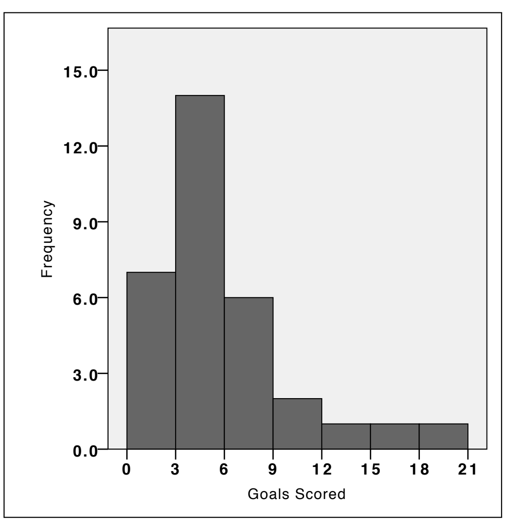
(a)



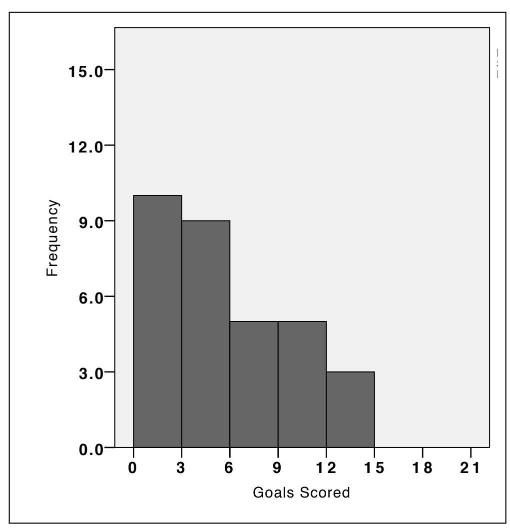
(b)



(c)



(d)



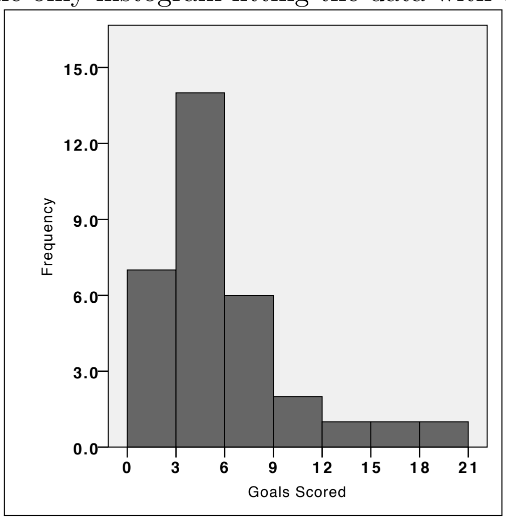
(e) None of the above

Solution:

The number of observations in the given intervals is shown below:

Interval	Frequency
$[0, 3)$	7
$[3, 6)$	14
$[6, 9)$	6
$[9, 12)$	2
$[12, 15)$	1
$[15, 18)$	1
$[18, 21)$	1

The only histogram fitting the data with this or any other configuration of intervals is



15. (5 pts.) A sample of 10 students were asked how many text books they bought for the Fall semester and the results are shown in the table below:

No. of Books	Frequency
2	1
4	2
5	4
6	2
8	1

The sample average is $\bar{x} = 5$, what is the **sample** standard deviation (rounded to two decimal places)?

(a) $s = 2.44$ (b) $s = 1.56$ (c) $s = 3.1$

(d) $s = 1.1$ (e) $s = 2.12$

Solution:

o_i	f_i	$o_i f_i$	$o_i - \bar{x}$	$(o_i - \bar{x})^2$	$(o_i - \bar{x})^2 f_i$
2	1	2	-3	9	9
4	2	8	-1	1	2
5	4	20	0	0	0
6	2	12	1	1	2
8	1	8	3	9	9
	$n = 10$	$\bar{x} = 50/10$ =5			$s^2 = 22/9$ $s = \sqrt{22/9} \approx 1.56$

16. (5 pts.) An experiment consists of rolling a pair of fair six sided dice. Let X denote the product of the two numbers which appear on the uppermost faces. What is $P(X > 2)$?

(a) $\frac{35}{36}$ (b) $\frac{3}{36}$ (c) $\frac{1}{36}$

(d) $\frac{33}{36}$ (e) $\frac{24}{36}$

Solution:

$$\begin{matrix} \{(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6)\} \end{matrix}$$

In the above equally likely sample space, only three outcomes correspond to the event $X \leq 2$, namely those highlighted in red.

$$P(X > 2) = 1 - P(X \leq 2) = 1 - \frac{3}{36} = \frac{33}{36}.$$

17. (5 pts.) The rules of a carnival game are as follows:

- The player pays $1 to play the game.
- The player then rolls a fair six-sided die.
- If the number on the die is odd, the game is over and the player gets nothing back from the game attendant.
- If the number on the die is even, the player draws a card randomly from a standard deck of 52 cards.
- If the card drawn is an ace, the player receives $100 from the attendant, otherwise, the game is over and the player gets nothing back from the game attendant.

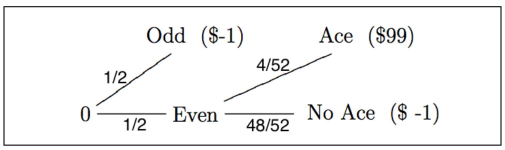
What are the expected earnings for the player in this game (correct to 2 decimal places)?

(a) $3.81 (b) $2.85 (c) $1.56

(d) −$2.34 (e) −$1.56

Solution:

the tree diagram shown below shows the player's net earnings at the end of each path.



Thus the probability distribution for the player's earnings is given by

Earnings	Prob.
-1	$\frac{1}{2}+\frac{1}{2}\cdot\frac{48}{52}=\frac{100}{104}$
99	$\frac{1}{2}\cdot\frac{4}{52}=\frac{4}{104}$

$$\text{expected earnings} = 99\cdot\frac{4}{104}-\frac{100}{104}=\frac{296}{104}\approx \$2.85.$$

18. (5 pts.) Hilary, who was very busy with Rowing Club activities, forgot to study for her finite math quiz. There are 4 multiple choice questions on the quiz and each question has 5 options for the answer. Hilary decides to randomly guess the answer to each question. What is the probability that Hilary will answer at least two questions correctly?

(a) $1-\left[(0.8)^4+4(0.2)(0.8)^3\right]$ (b) $(0.8)^4+4(0.2)(0.8)^3$

(c) $6(0.2)^2(0.8)^2$ (d) $1-6(0.2)^2(0.8)^2$

(e) $1-\left[(0.8)^4+4(0.2)(0.8)^3+6(0.2)^2(0.8)^2\right]$

Solution:

Let X denote the number of questions answered correctly. X is a binomial random variable with $n=4$ and $p=0.2$. $P(X\geq 2)=1-\left[P(X=0)+P(X=1)\right]=1-\left[(0.8)^4+4(0.2)(0.8)^3\right]$.

19. (5 pts.) Ten percent of the very large population of Medialand carry the LOL gene, strongly associated with random involuntary outbursts of laughter in those who carry the gene. Let X denote the number of people in a random sample of size 20 chosen from the population of Medialand who carry the LOL gene. Which of the following gives the expected value and standard deviation of X?

(a) $E(X) = 2,\ \sigma(X) = \sqrt{\frac{9}{10}}$

(b) $E(X) = 2,\ \sigma(X) = \sqrt{2}$

(c) $E(X) = 2,\ \sigma(X) = \sqrt{\frac{18}{10}}$

(d) $E(X) = \sqrt{2},\ \sigma(X) = \sqrt{\frac{9}{10}}$

(e) $E(X) = \sqrt{2},\ \sigma(X) = \sqrt{\frac{18}{10}}$

Solution:

X is a binomial random variable with $n = 20$ and $p = .1$. $E(X) = np = 2$ and $\sigma(X) = \sqrt{npq} = \sqrt{\frac{18}{10}}$.

20. (5 pts.) The lifetime (measured in miles covered) of car tires made by the Bad Year Tire Company is normally distributed with mean $\mu = 30,000$ miles and standard deviation $\sigma = 800$ miles. What is the probability that a tire chosen at random from those made by the Bad Year Tire Company will have a lifetime greater than $32,000$ miles?
(Tables for the standard normal distribution are attached at the end of your exam.)

(a) 0.0014

(b) 0.9938

(c) 0.9988

(d) 0.0062

(e) 0.1587

Solution:

$$P(X \geq 32,000) = P(Z \geq \frac{32,000 - 30,000}{800}) = P(Z \geq 2.5) = 1 - P(Z \leq 2.5)$$
$$= 1 - 0.9938 = 0.0062,$$

since Z is a standard normal random variable.

21. (5 pts.) Find the area under the standard normal curve between $Z = -1$ and $Z = 3.5$.

(a) 0.8411 (b) 0.9938 (c) 0.1587

(d) 0.9988 (e) 0.6154

Solution:

$$P(-1 \leq Z \leq 3.5) = P(Z \leq 3.5) - P(Z \leq -1) = 0.9998 - 0.1587 = 0.8411$$

22. (5 pts.) The scores for a standardized test given in Florin in 1987 were normally distributed with mean 110 and standard deviation 15. What percentage of the scores were more than 2.5 standard deviations away from the mean?

(a) 0.0062 (b) 0.9938 (c) 0.0668

(d) 0.1336 (e) 0.0124

Solution:
This is the probability that the Z-scores are either less than -2.5 or greater than 2.5. The Z scores have a standard normal distribution.

$$P(Z < -2.5) + P(Z > 2.5) = P(Z < -2.5) + [1 - P(Z < 2.5)]$$
$$= .0062 + 1 - .9938 = .0124.$$

23. (5 pts.) Ralph (R) and Connor (C) play a game where each one shows a number on a four sided die (with sides labelled 1, 2, 3 and 4) simultaneously. If the product of the numbers is even, Connor pays Ralph an amount equal to the sum of the two numbers shown. If the product of the numbers is odd, Ralph pays Connor an amount equal to the product of the numbers shown. Which of the following gives the payoff matrix (for R, with R as the row player) for this zero-sum game?

(a)

	1	2	3	4
1	1	−3	3	−5
2	−3	−4	−5	−6
3	3	−5	9	−7
4	−5	−6	−7	−8

(b)

	1	2	3	4
1	−1	2	−3	4
2	2	4	6	8
3	−3	6	−9	12
4	4	8	12	16

(c)

	1	2	3	4
1	−2	3	−4	5
2	3	4	5	6
3	−4	5	−6	7
4	5	6	7	8

(d)

	1	2	3	4
1	1	−2	3	−4
2	−2	−4	−6	−8
3	3	−6	9	−12
4	−4	−8	−12	−16

(e)

	1	2	3	4
1	−1	3	−3	5
2	3	4	5	6
3	−3	5	−9	7
4	5	6	7	8

Solution:

The payoff for R for the combinations given below are enough to eliminate all answers except for the given payoff matrix

Play	Payoff for R
R1C1	-1
R1C2	3

	1	2	3	4
1	−1	3	−3	5
2	3	4	5	6
3	−3	5	−9	7
4	5	6	7	8

You should check the other entries in the matrix to verify that it is the payoff matrix.

24. (5 pts.) The following matrix is the payoff matrix for the row player in a zero-sum game:

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

Which of the following statements is true?

(a) There are no saddle points in this matrix.

(b) There are three saddle points in this matrix.

(c) This game is strictly determined with a value of 1.

(d) This optimal strategy for the row player in this game is to always play Row 3.

(e) This game is strictly determined with a value of 2.

Solution:

				Min.
	(1)	2	2	1
	0	2	1	0
	−1	−1	1	−1
Max.	1	2	2	

This game has a unique saddle point at R1C1, thus the game is strictly determined, the optimal strategy for R is to always play R1 and the value of the game is $\nu = 1$.

25. (5 pts.) Ciall (The Column player) and Rory (The Row player) play a zero-sum game, with payoff matrix for Rory given by

$$\begin{bmatrix} 2 & 0 & 4 \\ 0 & -1 & 3 \\ 1 & 2 & -1 \end{bmatrix}$$

If Rory plays the mixed strategy [.2 .7 .1] and Ciall plays the mixed strategy $\begin{bmatrix} .2 \\ .5 \\ .3 \end{bmatrix}$, what is the expected payoff for Rory for the game?

(a) 2.51 (b) 0.69 (c) 3.83

(d) 0.86 (e) 1.12

Solution:

$$\begin{bmatrix} .2\ .7\ .1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 4 \\ 0 & -1 & 3 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} .2 \\ .5 \\ .3 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 & 2.8 \end{bmatrix} \begin{bmatrix} .2 \\ .5 \\ .3 \end{bmatrix} = [0.69]$$

26. (5 pts.) Chandler (The Column player) and Ross (The Row player) play a zero-sum game, with payoff matrix for Ross given by

$$\begin{bmatrix} -1 & 2 \\ 3 & -2 \end{bmatrix}$$

If Ross always plays the mixed strategy [.3 .7], Which of the following gives the best counterstrategy for Chandler?

(a) $\begin{bmatrix} .2 \\ .5 \end{bmatrix}$, (b) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} .3 \\ .7 \end{bmatrix}$

(d) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (e) $\begin{bmatrix} .5 \\ .5 \end{bmatrix}$

Solution:

The column player can always achieve the minimum payoff for R using a fixed strategy, so among those shown above we need only check the two fixed strategies for C to find which gives the least expected payoff for R.

$$[.3\ .7] \begin{bmatrix} -1 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.8 & -.8 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [-.8]$$

$$[.3\ .7] \begin{bmatrix} -1 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.8 & -.8 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = [1.8].$$

Therefore $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is the best counterstrategy for C.

27. (5 pts.) Carol (The Column player) and Raymond (The Row player) play a zero-sum game. The pay-off matrix is a two by two matrix. If we denote Raymond's strategy by $[p\ 1-p]$, the equations of the strategy lines corresponding to Carol's mixed strategies are given by

$$y = 2 - 4p \quad \text{and} \quad y = p - 2.$$

Which of the following gives Raymond's optimal mixed strategy?

(a) $[\ .6 \quad .4\ ]$ (b) $[\ .8 \quad .2\ ]$ (c) $[\ .2 \quad .8\ ]$

(d) $[\ .4 \quad .6\ ]$ (e) $[\ .5 \quad .5\ ]$

Solution:

We find the optimal mixed strategy for R where these two lines meet (if they meet in the interval $0 \le p \le 1$.)

$$y = 2 - 4p \text{ meets } y = p - 2$$

when $2 - 4p = p - 2$ or $4 = 5p$ or $p = 4/5 = .8$. Thus Raymond's best strategy is [.8 .2].

28. (5 pts.) Carlos (C) and Rosita (R) play a zero-sum game, with payoff matrix for Rosita given by

	C_1	C_2
R_1	1	5
R_2	7	2

What is Rosita's optimal mixed strategy for the game?
Note: The formulas given at the end of the exam may help.

(a) $\left[\ \frac{5}{9} \quad \frac{4}{9}\ \right]$ (b) $\left[\ \frac{1}{3} \quad \frac{2}{3}\ \right]$ (c) $\left[\ \frac{2}{3} \quad \frac{1}{3}\ \right]$

(d) $\left[\ \frac{4}{9} \quad \frac{5}{9}\ \right]$ (e) $\left[\ \frac{1}{2} \quad \frac{1}{2}\ \right]$

Solution:

From the formulas, if the payoff matrix is

	C_1	C_2
R_1	a	b
R_2	c	d

The optimal mixed strategy for R is given by $[p\ 1-p]$ where

$$p = \frac{d-c}{(a+d)-(b+c)} = \frac{2-7}{3-12} = \frac{-5}{-9} = \frac{5}{9}.$$

Therefore the optimal strategy for R is $\left[\ \frac{5}{9} \quad \frac{4}{9}\ \right]$.

29. (5 pts.) Cinderella (C) and Rapunzel (R) play a zero-sum game, with payoff matrix for Rapunzel given by

	C_1	C_2
R_1	1	3
R_2	4	1

Which of the following statements is true?
Note: The formulas given at the end of the exam may help.

(a) This is a strictly determined game.

(b) This is a fair game.

(c) The value of this game is $\nu = \frac{11}{5}$

(d) The value of this game is $\nu = 1$

(e) If both player's play their optimal strategies for this game, Cinderella's expected payoff is $\frac{1}{3}$.

Solution:

The game is not strictly determined since there is no entry which is a minimum in its row and a maximum in its column. By the formula, the value of the game is

$$\nu = \frac{ad - bc}{(a+d) - (b+c)} = \frac{1 - 12}{2 - 7} = \frac{-11}{-5} = \frac{11}{5}.$$

Therefore it is not a fair game. If both player's play their optimal strategies for this game, Cinderella's expected payoff is $\frac{-11}{5}$.

30. (5 pts.) Charlie (C) and Ruth (R) play a zero-sum game, with payoff matrix for Ruth given by

	C_1	C_2	C_3
R_1	2	5	1
R_2	7	2	3
R_3	7	−1	−1

Which of the following gives the optimal strategy for Ruth for this game?
Hint: You may need to reduce this matrix before applying the formulas given at the end of the exam.

(a) $\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{3}{5} & \frac{2}{5} & 0 \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{5} & 0 & \frac{4}{5} \end{bmatrix}$

(d) $\begin{bmatrix} \frac{1}{5} & \frac{4}{5} & 0 \end{bmatrix}$ (e) $\begin{bmatrix} \frac{4}{5} & \frac{1}{5} & 0 \end{bmatrix}$

Solution:

We see that Column C_3 dominates Column C_1 and thus we can reduce the matrix by removing Column 1 to get

	C_2	C_3
R_1	5	1
R_2	2	3
R_3	−1	−1

The row R_2 dominates the row R_3 in the new matrix, hence we can further reduce the matrix by removing it to get the fully reduced matrix

	C_2	C_3
R_1	5	1
R_2	2	3

Now, we can use the formula to determine R's optimal strategy as

$$\frac{3-2}{(5+3)-(2+1)} = \frac{1}{5}.$$

Thus R's optimal mixed strategy is

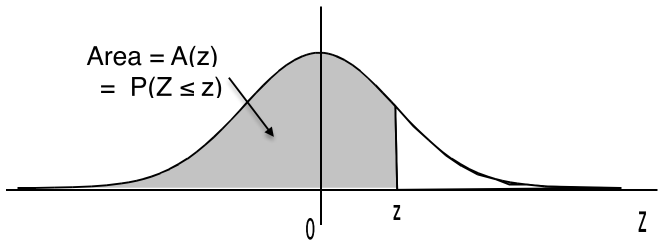
$$\begin{bmatrix} \frac{1}{5} & \frac{4}{5} & 0 \end{bmatrix}$$

For payoff matrix

	C_1	C_2
R_1	a	b
R_2	c	d

$$p = \frac{d-c}{(a+d)-(b+c)}$$
$$q = \frac{d-b}{(a+d)-(b+c)}$$
$$\nu = \frac{ad-bc}{(a+d)-(b+c)}.$$

Areas under the Standard Normal Curve



z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$
−3.50	.0002	−2.00	.0228	−.50	.3085	1.00	.8413	2.50	.9938
−3.45	.0003	−1.95	.0256	−.45	.3264	1.05	.8531	2.55	.9946
−3.40	.0003	−1.90	.0287	−.40	.3446	1.10	.8643	2.60	.9953
−3.35	.0004	−1.85	.0322	−.35	.3632	1.15	.8749	2.65	.9960
−3.30	.0005	−1.80	.0359	−.30	.3821	1.20	.8849	2.70	.9965
−3.25	.0006	−1.75	.0401	−.25	.4013	1.25	.8944	2.75	.9970
−3.20	.0007	−1.70	.0446	−.20	.4207	1.30	.9032	2.80	.9974
−3.15	.0008	−1.65	.0495	−.15	.4404	1.35	.9115	2.85	.9978
−3.10	.0010	−1.60	.0548	−.10	.4602	1.40	.9192	2.90	.9981
−3.05	.0011	−1.55	.0606	−.05	.4801	1.45	.9265	2.95	.9984
−3.00	.0013	−1.50	.0668	.00	.5000	1.50	.9332	3.00	.9987
−2.95	.0016	−1.45	.0735	.05	.5199	1.55	.9394	3.05	.9989
−2.90	.0019	−1.40	.0808	.10	.5398	1.60	.9452	3.10	.9990
−2.85	.0022	−1.35	.0885	.15	.5596	1.65	.9505	3.15	.9992
−2.80	.0026	−1.30	.0968	.20	.5793	1.70	.9554	3.20	.9993
−2.75	.0030	−1.25	.1056	.25	.5987	1.75	.9599	3.25	.9994
−2.70	.0035	−1.20	.1151	.30	.6179	1.80	.9641	3.30	.9995
−2.65	.0040	−1.15	.1251	.35	.6368	1.85	.9678	3.35	.9996
−2.60	.0047	−1.10	.1357	.40	.6554	1.90	.9713	3.40	.9997
−2.55	.0054	−1.05	.1469	.45	.6736	1.95	.9744	3.45	.9997
−2.50	.0062	−1.00	.1587	.50	.6915	2.00	.9772	3.50	.9998
−2.45	.0071	−.95	.1711	.55	.7088	2.05	.9798		
−2.40	.0082	−.90	.1841	.60	.7257	2.10	.9821		
−2.35	.0094	−.85	.1977	.65	.7422	2.15	.9842		
−2.30	.0107	−.80	.2119	.70	.7580	2.20	.9861		
−2.25	.0122	−.75	.2266	.75	.7734	2.25	.9878		
−2.20	.0139	−.70	.2420	.80	.7881	2.30	.9893		
−2.15	.0158	−.65	.2578	.85	.8023	2.35	.9906		
−2.10	.0179	−.60	.2743	.90	.8159	2.40	.9918		
−2.05	.0202	−.55	.2912	.95	.8289	2.45	.9929		