Counting, adding, multiplying, dividing

Math 10120, Spring 2014

January 31, 2014

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  - multiplication principle
  - addition principle
  - overcount principle

The multiplication principle

Suppose a process has two consecutive steps, with

- m choices for the first step, and
- n choices for the second (REGARDLESS OF FIRST STEP).

Then the total number of possible outcomes for the process is

$$mn$$

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Suppose a process has t consecutive steps, with

- m_1 choices for the first step,
- m_2 choices for the second (REGARDLESS OF FIRST STEP),
- m_3 choices for the third (REGARDLESS OF FIRST TWO STEPS),
- $\ldots$, and
- m_t choices for the tth (REGARDLESS OF EARLIER STEPS).

Then the total number of possible outcomes for the process is

$$m_1 m_2 m_3 \ldots m_t$$

The sum principle

Suppose at the beginning of an experiment you have to choose between one of two options, with

- m outcomes if you choose the first option, or
- n outcomes if you choose the second.

Then the total number of possible outcomes for the experiment is

$$m + n$$

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Suppose at the beginning of an experiment you have to choose between one of t options, with

- m_1 outcomes if you choose the first option,
- m_2 outcomes if you choose the second,
- ..., or
- m_t outcomes if you choose the tth.

Then the total number of possible outcomes for the experiment is

$$m_1 + m_2 + \ldots + m_t$$

The bottom line

If you have to do *A* **and then** *B*, and there are always the same number of ways of doing *B*, no matter what you did for *A*: MULTIPLY!

- There are five restaurants in town, and eight movies showing. I want to eat, **and then** go to a movie. I have a total of

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The overcount principle

Suppose you make an initial, naive, count of the elements of a set, and you get $ANSWER1$, but then you realize that you have counted each element of the set $OVERCOUNTFACTOR$ number of times. Then the real number of elements in the set is

$$\frac{ANSWER1}{OVERCOUNTFACTOR}$$

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- How many different anagrams does the word MUUMUU have?

$$\frac{6!}{2!4!} = 15$$