Section:______ **Name:** ________________________________ April 2, 2026

Linear Algebra and Differential Equations Tutorial
Quiz 8

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1. (2 points) Consider the following vectors:

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

They form a basis for $\mathbb{R}^2$. Apply the Gram-Schmidt process to obtain an orthogonal basis (You must use Gram-Schmidt for full credit).

Solution:

Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the orthogonal basis given by the Gram-Schmidt process. Then,

$$\mathbf{v}_1 = \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

and

$$\mathbf{v}_2 = \mathbf{u}_2 - \frac{\mathbf{u}_1 \cdot \mathbf{u}_2}{\mathbf{u}_1 \cdot \mathbf{u}_1}\mathbf{u}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3/2 \\ 3/2 \end{bmatrix}$$

2. (2 points) Find a least squares solution of $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$$

Solution:

The least squares solution is the vector $\hat{\mathbf{x}}$ that is the solution to the normal equations

$$A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$$

We have $A^TA = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ and $A^T\mathbf{b} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$. Hence, we see that

$$\hat{\mathbf{x}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Problem	Score
1	/ 2
2	/ 2
Total	/ 4