

Linear Algebra and Differential Equations Tutorial
Quiz 7

1. (2 points) Is the following set of vectors an orthogonal set? If so, is it an orthogonal basis for \mathbb{R}^3 ? Justify your answers.

$$\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$$

Solution:

Let v_1, v_2 , and v_3 be the vectors in the set. First, we check if the vectors are pairwise orthogonal by calculating their dot products:

$$v_1 \cdot v_2 = (2)(0) + (-1)(0) + (1)(0) = 0$$

$$v_1 \cdot v_3 = (2)(1) + (-1)(1) + (1)(-1) = 2 - 1 - 1 = 0$$

$$v_2 \cdot v_3 = (0)(1) + (0)(1) + (0)(-1) = 0$$

Since the dot product of every pair of distinct vectors is zero, **yes, it is an orthogonal set.**

Because the set contains the zero vector (v_2), it is a linearly dependent set. Therefore, it is not a basis for \mathbb{R}^3 , meaning **it is not an orthogonal basis** for \mathbb{R}^3 .

2. (2 points) Let $\mathbf{u} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$.

- (a) Find $\mathbf{u} \cdot \mathbf{v}$.

Solution:

$$\mathbf{u} \cdot \mathbf{v} = (3)(1) + (4)(0) + (0)(-1) = 3.$$

- (b) Find a unit vector in the direction of \mathbf{u} .

Solution:

A unit vector in the direction of \mathbf{u} is given by $\frac{\mathbf{u}}{\|\mathbf{u}\|}$, where $\|\mathbf{u}\| = \sqrt{3^2 + 4^2 + 0^2} = \sqrt{25} = 5$. Hence,

$$\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{1}{5} \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 3/5 \\ 4/5 \\ 0 \end{bmatrix}$$

- (c) Find $\text{proj}_{\mathbf{u}} \mathbf{v}$ (the orthogonal projection of \mathbf{v} onto \mathbf{u}).

Solution:

$$\text{proj}_{\mathbf{u}} \mathbf{v} = \frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{u} \cdot \mathbf{u}} \mathbf{u} = \frac{3}{25} \mathbf{u} = \begin{bmatrix} 9/25 \\ 12/25 \\ 0 \end{bmatrix}$$

Problem	Score
1	/ 2
2	/ 2
Total	/ 4