Section:______ **Name:** ________________________________ March 19, 2026

Linear Algebra and Differential Equations Tutorial
Quiz 6

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1. (2 points) Let $A = \begin{bmatrix} -3 & 12 \\ -2 & 7 \end{bmatrix}$. Find all the eigenvalues of the matrix A. Is A diagonalizable? Justify your answer.

Solution:

To find the eigenvalues we need to solve the equation $\det(A - \lambda I) = 0$:

$$\det \begin{bmatrix} -3-\lambda & 12 \\ -2 & 7-\lambda \end{bmatrix} = 0$$

Calculate the determinant:

$$\begin{aligned}(-3-\lambda)(7-\lambda) - (12)(-2) &= -21 + 3\lambda - 7\lambda + \lambda^2 + 24 \\ &= \lambda^2 - 4\lambda + 3 \\ &= (\lambda - 1)(\lambda - 3) = 0\end{aligned}$$

The eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = 3$.

Is A diagonalizable? Yes, A is diagonalizable. Because A is a 2×2 matrix and has 2 distinct real eigenvalues, it is guaranteed to have a basis of two linearly independent eigenvectors. Therefore, it can be diagonalized.

2. (2 points) Let $A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$. Find all the eigenvalues of the matrix A. Is A diagonalizable? Justify your answer.

Solution:

To find the eigenvalues we solve the equation $\det(A - \lambda I) = 0$:

$$\det \begin{bmatrix} 3-\lambda & -1 \\ 2 & 1-\lambda \end{bmatrix} = 0$$

Hence

$$\begin{aligned}(3-\lambda)(1-\lambda) - (-1)(2) &= 3 - 3\lambda - \lambda + \lambda^2 + 2 \\ &= \lambda^2 - 4\lambda + 5\end{aligned}$$

To find the roots, we use the quadratic formula:

$$\lambda = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i$$

The eigenvalues are complex: $\lambda_1 = 2 + i$ and $\lambda_2 = 2 - i$.

Is A diagonalizable? If we are strictly considering matrices over the real numbers ($\mathbb{R}$), A is **not diagonalizable** because it does not have real eigenvalues (and therefore no real eigenvectors). However, if we are working over the complex numbers ($\mathbb{C}$), A **is diagonalizable** because it has two distinct complex eigenvalues.

Problem	Score
1	/ 2
2	/ 2
Total	/ 4