Section:______ **Name:** ________________________________ February 26, 2026

Linear Algebra and Differential Equations Tutorial
Quiz 5

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1. (2 points) Let $T : \mathbb{R}^8 \to \mathbb{R}^{12}$ be a linear transformation which is one-to-one. What is the dimension of the range of T?

Solution: Since T is one-to-one, $\ker(T) = \{0\}$. By the Rank–Nullity Theorem,

$$\dim(\mathbb{R}^8) = \dim(\ker T) + \dim(\operatorname{range} T).$$

Thus,

$$8 = 0 + \dim(\operatorname{range} T),$$

so

$$\dim(\operatorname{range} T) = 8.$$

2. (2 points) A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ has outputs

$$T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} -1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Compute

$$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$$

Solution: Note that

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix} \right\}$$

is a basis of $\mathbb{R}^2$ (the 2×2 matrix with those vectors as columns has non zero determinant).

We express $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ in this basis. Solve

$$a \begin{bmatrix} 3 \\ 1 \end{bmatrix} + b \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

which gives $a = \dfrac{2}{5}$ and $b = \dfrac{1}{5}$. Hence

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{2}{5} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \frac{1}{5} \begin{bmatrix} -1 \\ 3 \end{bmatrix}.$$

By linearity of T,

$$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \frac{2}{5} T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) + \frac{1}{5} T\left(\begin{bmatrix} -1 \\ 3 \end{bmatrix}\right)$$

$$= \frac{2}{5} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \frac{1}{5} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4/5 \\ 1 \end{bmatrix}.$$

Problem	Score
1	/ 2
2	/ 2
Total	/ 4