Section:______ **Name:** ________________________________ February 19, 2026

Linear Algebra and Differential Equations Tutorial
Quiz 4

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1. (2 points) Find the change of basis matrix $P_{C\leftarrow B}$ where the bases are

$$B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\} \quad \text{and} \quad C = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \end{bmatrix} \right\}.$$

Solution: You can do this by doing Gauss-Jordan elimination, by computing the product $C^{-1} \cdot B$, or by writing the vectors in the basis B as a linear combination of the vectors in C.

I only do Gauss-Jordan elimination under death threats, and in the case of 2×2 matrices, computing the product $C^{-1} \cdot B$ is quite fast.

$$C^{-1} = \frac{1}{4-3} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \implies P_{C\leftarrow B} = C^{-1}B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 11 \\ 5 & 7 \end{bmatrix}$$

2. (2 points) Are the polynomials $\{1 + x, x + x^2\}$ linearly independent in $\mathscr{P}_2$, the real vector space of polynomials of degree at most 2?

Solution: Yes, they are independent. Recall that two vectors v_1, v_2 are linearly independent if whenever $c_1v_1 + c_2v_2 = 0$, it must be the case that $c_1 = c_2 = 0$. So, let

$$c_1(1 + x) + c_2(x + x^2) = 0.$$

Factoring the coefficients, $c_1 + (c_1 + c_2)x + c_2x^2 = 0$. This implies that $c_1 = 0$, $c_1 + c_2 = 0$ and $c_2 = 0$ and so the polynomials are linearly independent.

Another way to see why (although not as rigorous as the argument above) is because two vectors are linearly dependent if and only if one is a scalar multiple of the other, and it is clear to see that $c(1 + x) \neq x + x^2$. **Beware:** $c = x$ does not count, because x is a variable and c must be some real number.

Problem	Score
1	/ 2
2	/ 2
Total	/ 4