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Linear Algebra and Differential Equations Tutorial
Quiz 3

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1. (2 points) Determine if the following vectors are linearly independent.

$$\begin{bmatrix}1\\3\\1\\2\end{bmatrix} \quad \begin{bmatrix}1\\2\\1\\3\end{bmatrix} \quad \begin{bmatrix}2\\5\\2\\5\end{bmatrix}$$

Solution: Form the matrix with the given vectors as columns:

$$\begin{bmatrix}1&1&2\\3&2&5\\1&1&2\\2&3&5\end{bmatrix}.$$

Row reduce:

$$\begin{bmatrix}1&1&2\\3&2&5\\1&1&2\\2&3&5\end{bmatrix} \xrightarrow{R_2\leftarrow R_2-3R_1} \begin{bmatrix}1&1&2\\0&-1&-1\\1&1&2\\2&3&5\end{bmatrix} \xrightarrow{R_3\leftarrow R_3-R_1} \begin{bmatrix}1&1&2\\0&-1&-1\\0&0&0\\2&3&5\end{bmatrix}$$
$$\xrightarrow{R_4\leftarrow R_4-2R_1} \begin{bmatrix}1&1&2\\0&-1&-1\\0&0&0\\0&1&1\end{bmatrix} \xrightarrow{R_4\leftarrow R_4+R_2} \begin{bmatrix}1&1&2\\0&-1&-1\\0&0&0\\0&0&0\end{bmatrix}.$$

There are only 2 pivot points, but there are 3 columns. One of the columns does not have a pivot point. So the column vectors are *not* linearly independent, or in other words, linearly dependent.

Solution: Another way to solve:

Let

$$v_1 = \begin{bmatrix}1\\3\\1\\2\end{bmatrix}, \quad v_2 = \begin{bmatrix}1\\2\\1\\3\end{bmatrix}, \quad v_3 = \begin{bmatrix}2\\5\\2\\5\end{bmatrix}.$$

Notice that

$$v_1 + v_2 = \begin{bmatrix}1\\3\\1\\2\end{bmatrix} + \begin{bmatrix}1\\2\\1\\3\end{bmatrix} = \begin{bmatrix}2\\5\\2\\5\end{bmatrix} = v_3.$$

So $v_3 = v_1 + v_2$ is a nontrivial linear relation between v_1, v_2, v_3, and so the vectors are linearly dependent.

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2. (2 points) Consider the matrix $A = \begin{bmatrix} 1 & -5 \\ 0 & -1 \\ -2 & 2 \end{bmatrix}$. Write **three distinct vectors** in the column space of A.

Solution: The column space of A is $\text{Col}(A) = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix} \right\}$. For example, three distinct vectors in $\text{Col}(A)$ are

$$\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} -4 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + \begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}.$$

Problem	Score
1	/ 2
2	/ 2
Total	/ 4