

Linear Algebra and Differential Equations Tutorial
Quiz 02

1. (2 points) For each one of the following matrices, determine whether the matrix is invertible, and find its inverse if it exists. Show all work.

(a) $\begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 11 & 3 \\ -3 & -1 \end{bmatrix}$

Solution: A 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible if $ad - bc \neq 0$ and the inverse is

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

(a) The determinant is $2(6) - (-4)(-3) = 0$, so the matrix is not invertible.

(b) The determinant is $-11 + 9 = -2$, so the matrix is invertible and the inverse is

$$A^{-1} = -\frac{1}{2} \begin{bmatrix} -1 & -3 \\ 3 & 11 \end{bmatrix} = \begin{bmatrix} 1/2 & 3/2 \\ -3/2 & -11/2 \end{bmatrix}$$

2. (2 points) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be defined by

$$T \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} x + 2z \\ 2 \end{bmatrix}.$$

Determine whether T is a linear transformation. Clearly justify your answer to receive full credit.

Solution: This is **not** a linear transformation for several reasons. A quick one to check is that a linear transformation must send the zero vector to the zero vector, whereas

$$T \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 2 \end{bmatrix},$$

so T cannot be a linear transformation.

Problem	Score
1	/ 2
2	/ 2
Total	/ 4