Section:_____ **Name:** _____ April 23, 2026

Linear Algebra and Differential Equations Tutorial
Quiz 10

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1. (2 points) Consider the equation

$$y' = (y^3 - 4y)(9 - y^2)$$

with initial value $y(-1) = 1$. Assuming $\lim_{t\to\infty} y(t)$ exists, find it without solving the equation.

Solution:

The constant solutions are $y = 0,\ y = \pm 2,\ y = \pm 3$. Since $y(-1) = 1$ the solution lies between the lines $y = 0$ and $y = 2$. Here $f(y) = (y^3 - 4y)(9 - y^2)$ is negative since $f(1) = -3 \cdot 8 = -24$. Hence y is decreasing so the limit is 0.

2. (2 points) If y is the solution to

$$y' + 4xy = x \quad \text{with} \quad y(1) = 2,$$

find $y(0)$.

Solution:

Integrating factor: $\mu = e^{2x^2}$ so we need

$$\int xe^{2x^2}\,dx = \frac{1}{4}e^{2x^2} + C$$

and the general solution is

$$y = \frac{1}{4} + Ce^{-2x^2}.$$

Then

$$y(1) = \frac{1}{4} + Ce^{-2} = 2$$

so

$$C = \frac{7e^2}{4}$$

and

$$y = \frac{1}{4} + \frac{7e^2}{4e^{2x^2}}.$$

Then

$$y(0) = \frac{1}{4} + \frac{7e^2}{4}.$$

Problem	Score
1	/ 2
2	/ 2
Total	/ 4