Name: ______________________________ January 22, 2026

Linear Algebra and Differential Equations Tutorial
Quiz 1

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1. (2 points) Determine whether the vector $\mathbf{w}$ can be written as a linear combination of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$. If yes, find scalars c_1, c_2, c_3 such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{w}$.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{w} = \begin{bmatrix} 7 \\ 3 \\ 5 \end{bmatrix}$$

Solution: We set up the augmented matrix corresponding to the system:

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 7 \\ 0 & 3 & 0 & 3 \\ -1 & -1 & 1 & 5 \end{array}\right]$$

Perform row operations to reach Reduced Row Echelon Form (RREF):

$$\xrightarrow{R_3 \leftarrow R_3 + R_1} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 7 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 1 & 12 \end{array}\right] \xrightarrow{R_2 \leftarrow \frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 7 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 12 \end{array}\right] \xrightarrow{R_1 \leftarrow R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 12 \end{array}\right]$$

Result: Reading the matrix, we find the unique solution:

$$\boxed{c_1 = 6, \quad c_2 = 1, \quad c_3 = 12}$$

2. (2 points) Determine the most general solution to the system of equations below.

$$4x_1 + 6x_2 = 10$$
$$3x_1 + 7x_2 = 0$$

Solution: We form the augmented matrix:

$$\left[\begin{array}{cc|c} 4 & 6 & 10 \\ 3 & 7 & 0 \end{array}\right]$$

Now we will do Gaussian elimination followed by back substitution to find the solution to the system.

$$\xrightarrow{R_1 \leftarrow \frac{1}{2}R_1} \left[\begin{array}{cc|c} 2 & 3 & 5 \\ 3 & 7 & 0 \end{array}\right] \xrightarrow{R_2 \leftarrow 2R_2 - 3R_1} \left[\begin{array}{cc|c} 2 & 3 & 5 \\ 0 & 5 & -15 \end{array}\right]$$

From row 2 ($5x_2 = -15$):

$$x_2 = \frac{-15}{5} = -3$$

Substitute $x_2 = -3$ into row 1 ($2x_1 + 3x_2 = 5$):

$$2x_1 + 3(-3) = 5 \implies x_1 = 7$$

Final Answer: The solution is unique and is

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \end{bmatrix}}$$