Math 20580 Name:____________________

Midterm 3 Instructor:____________________

April 16, 2026 Section:____________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an × through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1. a b c d e

2. a b c d e

3. a b c d e

4. a b c d e

5. a b c d e

6. a b c d e

7. a b c d e

8. a b c d e

Multiple Choice.

9.

10.

11.

12.

Total.

Part I: Multiple choice questions (7 points each)

1. Find the orthogonal projection of the vector $\vec{v} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ onto the vector $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

   (a) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1/2 \\ -1/2 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ (e) $\vec{0}$

2. If A is a 3×5 matrix of rank 2, what is the dimension of the orthogonal complement of the row space of A^T?

   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

3. Find the distance between $\vec{w} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ and the subspace $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$ (e) 3

4. Which of the following functions is the general solution of the equation $y' - 4y = 0$?

(a) $e^{-4x} + C$ (b) $4C + e^{x}$ (c) $e^{4x} - C$ (d) $e^{-x} - 4$ (e) $C \cdot e^{4x}$

5. Find the solution of the initial value problem

$$\begin{cases} y' + y = 2e^x, \\ y(0) = 0 \end{cases}$$

(a) e^x (b) $2e^x - e^{-x}$ (c) xe^x (d) $e^x - e^{-x}$ (e) $1 - e^{-x}$

6. Consider the initial value problem

$$\frac{dy}{dt} = y(y-3), \qquad y(0) = 1.$$

Which of the following describes the nature of the solution?

(a) $\lim_{t\to-\infty} y(t) = 3;$ $\lim_{t\to\infty} y(t) = 0$

(b) $\lim_{t\to-\infty} y(t) = 0;$ $\lim_{t\to\infty} y(t) = 3$

(c) $\lim_{t\to-\infty} y(t) = 3;$ $\lim_{t\to\infty} y(t) = \infty$

(d) $\lim_{t\to-\infty} y(t) = -\infty;$ $\lim_{t\to\infty} y(t) = 0$

(e) $\lim_{t\to-\infty} y(t) = 0;$ $\lim_{t\to\infty} y(t) = -\infty$

7. The differential equation

$$\frac{\partial y}{\partial t} + 2\frac{\partial y}{\partial x} = y + x$$

is

(a) an equation of order 2 (b) a partial differential equation (c) exact
(d) separable (e) an ordinary differential equation

8. Consider the orthogonal basis $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ of $\mathbb{R}^3$, where

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Find the coordinate vector $[\vec{w}]_{\mathcal{B}}$ if $\vec{w} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$.

(a) $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ (e) $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

Part II: Partial credit questions (11 points each). Show your work.

9. Let $W = \text{Span}\{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$, where

$$\vec{w}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{w}_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{w}_3 = \begin{bmatrix} 3 \\ 0 \\ 1 \\ -1 \end{bmatrix}.$$

(a) Apply the Gram-Schmidt process to find an orthonormal basis for W.

(b) Find the QR decomposition of the matrix A with columns $\vec{w}_1, \vec{w}_2, \vec{w}_3$.

10. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$.

(a) Find the least squares solution to the equation $A\vec{x} = \vec{b}$.

(b) Find the vector in the column space of A which is closest to $\vec{b}$.

11. Consider the differential equation

$$(3xy^2 + 4)\,dx + 2x^2y\,dy = 0.$$

(a) Explain why the equation is not exact.

(b) Find an integrating factor μ which only depends on the variable x.

(c) Determine the explicit solution which satisfies the initial condition $y(1) = 1$.

12. (a) Find the solution of the initial value problem

$$\begin{cases} \dfrac{dy}{dx} - 2xy^2 = 0 \\ y(0) = 1 \end{cases}$$

(b) Find the maximal interval on which the solution in (a) is defined.