Math 20580 Name:____________________
Midterm 2 Instructor:____________________
March 5, 2026 Section:____________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an × through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1. a b c d e
2. a b c d e
3. a b c d e
4. a b c d e
5. a b c d e
6. a b c d e
7. a b c d e
8. a b c d e

Multiple Choice.

9.

10.

11.

12.

Total.

Part I: Multiple choice questions (7 points each)

1. Suppose that A is a 2×2 matrix such that $\det(A) = -3$, and that $B = \begin{bmatrix} 7 & -15 \\ 23 & -121 \end{bmatrix}$.
What is $\det(2BA^TB^{-1})$?

(a) -12 (b) -121 (c) 251 (d) -502 (e) 23

2. Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 0 & 3 \\ -1 & 2 & 4 \\ -3 & 0 & 8 \end{bmatrix}$$

(a) 0,0,2 (b) 2,2,8 (c) -3,2,3 (d) 2,5,5 (e) -1,2,3

3. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Find the $(1, 3)$-entry of A^{-1} (the entry in row 1 and column 3 of the inverse of A).

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

4. Find the area of the parallelogram whose vertices are

$$(0, -2), \quad (6, -1), \quad (-3, 1), \quad (3, 2).$$

(a) 6 (b) 15 (c) 21 (d) 3 (e) 12

5. Consider the subspace of $\mathcal{P}_3$ defined by

$$W = \text{Span}\left\{1 - x, x - x^2, x^3 - x^2, 1 - 2x + x^3, 1 - x + 2x^3\right\}.$$

What is the dimension of W?

(a) 5 (b) 4 (c) 3 (d) 2 (e) 1

6. Suppose that the system of equations

$$\begin{cases} (s-2)x_1 + 3x_2 = 5 \\ x_1 + (s-4)x_2 = 6 \end{cases}$$

has a unique solution. Find the value of x_2 in terms of the parameter s.

(a) $\dfrac{-6s+17}{s^2-6s+8}$ (b) $\dfrac{5s-38}{s^2-6s+8}$ (c) $\dfrac{5s-38}{s^2-6s+5}$ (d) $\dfrac{6s-17}{s^2-6s+8}$ (e) $\dfrac{6s-17}{s^2-6s+5}$

7. Recall that $\mathcal{P}_3$ is the vector space of polynomials of degree at most 3, and consider the linear transformation $T : \mathcal{P}_3 \longrightarrow \mathbb{R}$ defined by

$$T(p(x)) = p(1).$$

What is the dimension of the kernel of T?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

8. Which of the following sets is linearly independent in $\mathcal{P}_3$?

(I) $\{\,1 + 2x,\ 3 + 6x\,\}$

(II) $\{\,1,\ x,\ x^2\,\}$

(III) $\{\,x^3 + x,\ 2x^3 + 2x,\ x^2\,\}$

(IV) $\{\,x^2 + 1\,\}$

(a) II and IV only (b) I, II, III only (c) I and II only (d) III and IV only
(e) I, III, IV only

Part II: Partial credit questions (11 points each). Show your work.

9. Consider the bases

$$\mathcal{B} = \left\{x, 1 + x^2, x + x^2\right\} \qquad \text{and} \qquad \mathcal{C} = \{1, 1 + x, x^2\}$$

of $\mathcal{P}_2$ (the vector space of polynomials of degree at most 2 in the variable x).

(a) Find the change-of-basis matrix $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}$ from $\mathcal{C}$ to $\mathcal{B}$.

(b) Given the vector $p(x) = 4 - 2x + x^2$ in $\mathcal{P}_2$, find $[p(x)]_{\mathcal{B}}$ and $[p(x)]_{\mathcal{C}}$.

10. Consider the vector space $\mathcal{P}_2$ of polynomials of degree at most two, the vector space $M_{2,2}$ of 2×2 matrices, and the transformation $T : \mathcal{P}_2 \longrightarrow M_{2,2}$ given by

$$T(p(x)) = \begin{bmatrix} p(0) & p(1) \\ p'(1) & p'(0) \end{bmatrix},$$

where $p'(x)$ denotes the derivative of $p(x)$.

(a) Compute the matrix $T(a + bx + cx^2)$ explicitly in terms of a, b, c.

(b) Find the matrix of T relative to the standard basis $\mathcal{B} = \{1, x, x^2\}$ of $\mathcal{P}_2$ and the standard basis $\mathcal{C} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ of $M_{2,2}$.

11. Consider the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & t \\ 1 & 1 & t & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

where t is some real number.

(a) Calculate the determinant of A (your answer may depend on t).

(b) Find all values of t such that A is invertible.

12. Let A be the matrix

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}.$$

(a) Find the eigenvalues of A.

(b) Diagonalize A, that is, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

$P =$ ______________________ $D =$ ______________________