Math 20580 Name:________________________

Midterm 1 Instructor:________________________

February 12, 2026 Section:________________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1. a b c d e
2. a b c d e
3. a b c d e
4. a b c d e
5. a b c d e
6. a b c d e
7. a b c d e
8. a b c d e

Multiple Choice.

9.

10.

11.

12.

Total.

Part I: Multiple choice questions (7 points each)

1. Consider the linear system

$$\begin{cases} 3x + 2y = a \\ 5x + 4y = b \end{cases}$$

   If (x, y) is a solution, which of the following describes y in terms of a, b?

   (a) $y = 2a - b$ (b) $y = 6a + 8b$ (c) $y = (-5a + 3b)/2$ (d) $y = 2a + 5b$

   (e) y is not determined by a, b

2. Consider the vectors $\vec{v}_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$. Which of the following sets of vectors span $\mathbb{R}^2$?

   (I) $\{\vec{v}_1, \vec{v}_2\}$ (II) $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ (III) $\{\vec{v}_2\}$ (IV) $\{\vec{v}_2, \vec{v}_3\}$

   (a) III only (b) I and III only (c) I and II only (d) II and IV only

   (e) III and IV only

3. Let $T\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the linear map given by counterclockwise rotation of the plane about the origin by an angle of $30°$ (or $\dfrac{\pi}{6}$ in radians). Let A be the standard matrix of the transformation T. Which of the following matrices is equal to $A^3 = A \cdot A \cdot A$?

(a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

4. Let $\mathcal{P}_3$ denote the vector space of polynomials of degree at most 3,

$$\mathcal{P}_3 = \{p(x) = a_0 + a_1x + a_2x^2 + a_3x^3\}$$

Which among the following subsets of $\mathcal{P}_3$ is a subspace?

I. The polynomials $p(x)$ in $\mathcal{P}_3$ satisfying $p(0) = 1$.

II. The polynomials $p(x)$ in $\mathcal{P}_3$ satisfying $p(1) = 0$.

III. The polynomials $p(x)$ in $\mathcal{P}_3$ satisfying $a_1 \geq 1$ and $a_2 \leq 2$.

IV. The polynomials $p(x)$ in $\mathcal{P}_3$ that have degree at most two.

(a) III and IV only (b) IV only (c) I, III, and IV only

(d) II and IV only (e) III only

5. Given the matrices

$$A = \begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$$

determine $A^{-1}B - AB^T$.

(a) $\begin{bmatrix} 1 & 3 \\ -1 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} 4 & 2 \\ -6 & 0 \end{bmatrix}$

6. Consider the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{v}_3 = \begin{bmatrix} 1-t \\ 1+t \\ 1 \end{bmatrix}$$

For which value of t does the vector $\vec{v}_3$ belong to $\text{Span}(\vec{v}_1, \vec{v}_2)$?

(a) all $t \leq 2$ (b) $t = 2$ and $t = -1$ (c) $t = -3$ only (d) $t = 0$ only

(e) no value of t

7. Which of the following sets is linearly independent?

(I) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix} \right\}$ (II) $\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ (III) $\left\{ \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \right\}$ (IV) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$

(a) I, II, III only (b) II and IV only (c) I and II only

(d) III and IV only (e) I, III, IV only

8. Let A be a 8×7 matrix of rank 3. Which of the following is equal to the dimension of the null space of the transpose matrix A^T?

(a) 0 (b) 3 (c) 4 (d) 5 (e) 7

Part II: Partial credit questions (11 points each). Show your work.

9. Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 1 & -1 \\ 1 & 0 & 1 & 0 & 3 \end{bmatrix}$$

(a) Find a basis for Col(A) (the column space of A).

(b) Find a basis for Row(A) (the row space of A).

(c) Find a basis for Nul(A) (the null space of A).

10. Find the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

11. Consider the linear transformation $T\colon \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ defined by

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_2 + x_3 \end{bmatrix},$$

and the matrix transformation $S(\vec{x}) = A\vec{x}$, where

$$A = \begin{bmatrix} -3 & -1 \\ 3 & 2 \\ -2 & -2 \end{bmatrix}.$$

(a) Find the standard matrix of T.

(b) Find the standard matrices of the compositions $S \circ T$ and $T \circ S$.

(c) Find a vector $\vec{v}$ in $\mathbb{R}^3$ with $T(\vec{v}) = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$.

12. Consider the bases $\mathcal{B}$ and $\mathcal{C}$ of $\mathbb{R}^3$ given by

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 2 \end{bmatrix} \right\}, \qquad \mathcal{C} = \left\{ \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

(a) Find the change of coordinate matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ from $\mathcal{B}$ to $\mathcal{C}$ (recall that $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ is the matrix such that $[\vec{x}]_{\mathcal{C}} = \underset{\mathcal{C}\leftarrow\mathcal{B}}{P} \cdot [\vec{x}]_{\mathcal{B}}$ for all vectors $\vec{x}$ in $\mathbb{R}^3$).

(b) If $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, determine the coordinate vectors $[\vec{v}]_{\mathcal{B}}$ and $[\vec{v}]_{\mathcal{C}}$.