Worksheet 10

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Determine whether the series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$.

2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$.

3. $\displaystyle\sum_{n=1}^{\infty} \tan\left(\frac{1}{n^2+3n+4}\right)$.

4. Show that if $a_n > 0$ and $\lim_{n\to\infty} na_n \neq 0$, then $\sum a_n$ is divergent.

5. Show that if $a_n > 0$ and $\sum a_n$ is convergent, then $\sum \ln(1+a_n)$ is convergent.

6. Show that if $a_n > 0$ and $\sum a_n$ is convergent, then $\sum \sin(a_n)$ is convergent.

7. If $\sum a_n$ and $\sum b_n$ are both convergent series with positive terms, is it true that $\sum a_n b_n$ is also convergent?

8. (Cauchy condensation test) If $\{a_n\}$ is a monotone decreasing sequence, then $\sum a_n$ converges if and only if $\sum 2^n a_{2^n}$ converges.

9. Use the Cauchy condensation test to determine the values of p for which

$$\sum_{n=3}^{\infty} \frac{1}{n\ln(n)(\ln(\ln(n)))^p}$$

is convergent.