

A simultaneous generalization of Ramsey, Hindman, and Hales-Jewett's Theorems

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In recent years, the use of nonstandard models of the natural numbers and the tools of nonstandard analysis has proven fruitful in discovering and proving new results in Ramsey theory. In particular, the technique of iterated nonstandard extensions has proven very effective.

Van der Waerden's famous theorem of 1927 states that in any finite coloring (partition) of the natural numbers $\mathbb{N} = C_1 \cup \dots \cup C_r$, one can find arbitrarily long arithmetic progressions that are “monochromatic,” that is, included in a color C_j . Hales-Jewett's theorem of 1963 is a strong Ramsey property of the set of words W_A over a finite alphabet A , from which Van der Waerden's theorem can be directly derived as a particular case. Another now classic result of arithmetic Ramsey theory is Hindman's theorem of 1974, which states that for every finite coloring of $\mathbb{N}$ there exists an increasing sequence (x_n) such that all finite sums of distinct elements $x_{n_1} + \dots + x_{n_k}$ are monochromatic. Over the past 60 years, there has been intense research aimed to extend and generalize the above fundamental results of Ramsey theory.

After a brief introduction to nonstandard methods in combinatorics, in this talk I will illustrate the following new result (joint work with Renling Jin), obtained using nonstandard methods.

Theorem. *Let $W = W_A$ be the set of words over an alphabet A , let $B \subset W$ be a finite set, and let $k \in \mathbb{N}$. For every finite coloring of the k -tuples of words $[W]^k$ there exists a sequence of variable words $(w_n(v))_{n \in \mathbb{N}}$ of increasing lengths such that for every sequence $(b_n)_{n \in \mathbb{N}}$ in B and for all nonempty finite subsets $F_1 < F_2 < \dots < F_k$ of $\mathbb{N}$, the following k -tuples are monochromatic:*

$$\left\{ \prod_{i \in F_1} w_i(b_i), \prod_{i \in F_2} w_i(b_i), \dots, \prod_{i \in F_k} w_i(b_i) \right\}.$$

(If $F = \{n_1 < \dots < n_s\}$, we denote $\prod_{i \in F} w_i(b_i)$ the concatenation of the finite list of words $w_{n_1}(b_{n_1}), \dots, w_{n_s}(b_{n_s})$.)

I will show that, as particular cases of the above theorem, one obtains all four previously mentioned results, namely Ramsey's theorem, Hales-Jewett's theorem and hence Van der Waerden's theorem, and Hindman's theorem.

Reference: M. Di Nasso and R. Jin, *A simultaneous generalization of Ramsey, Hindman, and Hales-Jewett's Theorems*, 2026, submitted (arXiv:2607.24413).