My favorite proof of the Pythagorean Theorem

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The Pythagorean Theorem is the first nontrivial geometric fact taught to children:

Pythagorean Theorem. *Consider a right triangle whose hypotenuse has length c and whose other two sides have length a and b. Then $a^2 + b^2 = c^2$.*

There are an enormous number of proofs of this result. In this note, I will explain my favorite. I do not know a precise reference for it, but I have heard it attributed to Einstein.

Proof of Pythagorean Theorem. Let Z be the triangle in question. Divide Z into right triangles X and Y as follows:
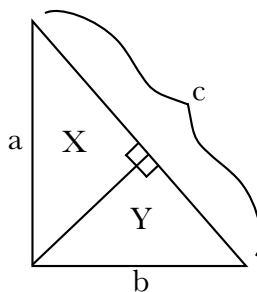
The angles of X and Y and Z are all the same, so all three triangles are similar. Let W be the triangle that is similar to these three triangles and whose hypotenuse has length 1. The hypotenuses of X and Y and Z are a and b and c, respectively. Since scaling a triangle by r scales its area by r^2, we have

$$\text{area}(X) = a^2 \text{ area}(W) \quad \text{and} \quad \text{area}(Y) = b^2 \text{ area}(W) \quad \text{and} \quad \text{area}(Z) = c^2 \text{ area}(W).$$

Plugging these into the formula

$$\text{area}(X) + \text{area}(Y) = \text{area}(Z),$$

we get

$$a^2 \text{ area}(W) + b^2 \text{ area}(W) = c^2 \text{ area}(W).$$

Dividing by $\text{area}(W)$, we deduce that $a^2 + b^2 = c^2$. $\qquad\square$

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